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Solution of Linearized Flat Problem of Hydrodynamics (IVF)

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***Abstract*. In the present paper a method of the generalized potential to planes is applied for the solution of the linearized according to Oseen of flat problem of hydrodynamics incompressible viscous fluid (IVF). Generalized potential simple layer containing McDonald function serves kernel for generalized potential to planes.**

For finding of an unknown density of the potential simple layer is received linear integral equation, containing double integral from curvilinear integral along border of the streamlined area.

Sharing the pressure is in turn defined by potential simple layer with density of the potential, determined by linear integral equation, hanging from solution specified above integral equation. The offered method of the successive iterations, allowing elaborate the solution of the problem before achievement given to accuracy.

As example of exhibit to theories is considered solution of the problem theory of hydrodynamic greasing

***Keywords* - incompressible viscous fluid, flat problem of hydrodynamics, problem theory of hydrodynamic greasing, linear integral equations, stream function.**

I

Consider problem of a flow of a contour l uniform flow (IVF) $\vec{u}(u_x; u_y)$. Choose the Cartesian system coordinates (XOY) with the centre O into domain D, bounded by contour l (Fig. 1.).

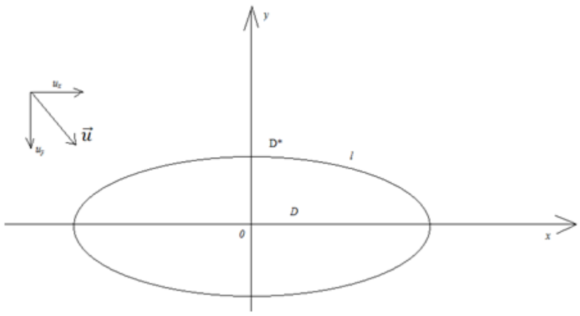

Fig. 1.

Navier – Stokes equations (NS) of flat problem (IVF)

$$(\vec{v}\vec{\nabla})\vec{v} = -1/\rho \cdot \vec{\nabla} p + \nu\Delta\vec{v}$$

$$div\,\vec{v} = 0. \quad (1)$$

Boundary conditions

$$\vec{v}\big|_l = 0, \vec{v} \to \vec{u}, \quad p \to p_0, \quad r \to \infty; \; r = \sqrt{x^2 + y^2} \quad (2)$$

Introduce non-dimensional variables

$$\vec{v} = u\overrightarrow{v^*}; \; u = \sqrt{u_x{}^2 + u_y{}^2};$$

$$\vec{u} = \vec{u} \;\; (\alpha, \beta); \; \alpha = \frac{u_x}{u}; \; \beta = \frac{u_y}{u} \quad (3)$$

$$p = \rho u^2 p^*, \vec{r} = d\overrightarrow{r^*}(x^*, y^*), \; d = diam(D)$$

Equation (1) and conditions (2) are written in non-dimensional variables (3)

$$\mathrm{Re}(\vec{v}\vec{\nabla})\vec{v} = -\vec{\nabla}p + \Delta\vec{v}, \; div\,\vec{v} = 0 \quad (4)$$

$$\vec{v}\big|_l = 0, \vec{v} \to \vec{u}(\alpha, \beta), \quad p \to p_1, \quad r \to \infty; \; p_1 = \frac{p_0}{\rho u^2}. \quad (5)$$

According to Oseen [1-3, 6-9, 12] execute linearization of the relation (4)

$$\mathrm{Re}(\vec{u}\vec{\nabla})\vec{v} = -\vec{\nabla}p + \Delta\vec{v}. \quad (6)$$

Introduce stream function

$$v_x = \frac{\partial\Psi}{\partial y}, v_y = -\frac{\partial\Psi}{\partial x}. \quad (7)$$

And rewrite (6), (7) in form of the generalized Helmholtz equation [4]

$$Re\left(\alpha\frac{\partial\Omega}{\partial y} + \beta\frac{\partial\Omega}{\partial x}\right) = \Delta\,\Omega; \quad (8)$$

$$\Omega = -\Delta\psi; \vec{\Omega} = (0,0,\Omega); \; \vec{\Omega} = \overrightarrow{rot}\,\vec{v}. \quad (9)$$

By supposing, that $\gamma = \frac{Re}{2}$

and $\Omega = e^{\gamma(\alpha x + \beta y)}\omega(x, y)$ (10)

obtain canonical form for (8)

$$\Delta\vec{\omega} = \gamma\vec{\omega} \quad (11)$$

McDonald function is a solution of the equation (11). This solution must depend that r only and $\lim\vec{\varpi} = 0$, if $r \to 0$. Therefore,

$$\omega = K_0(\gamma \cdot r); \quad K_0(r) = \int_1^{\infty} (e^{-\rho\xi} \cdot d\xi) \Big/ \sqrt{\xi^2 - 1} \quad (12)$$

Integral presentation of the solution of the linear equation (8) has form

$$\Omega(m) = \frac{1}{\pi} \int_l e^{\gamma q_{mp}} K_0(\gamma r_{mp}) \cdot \mu(p) \cdot dlp; \quad (13)$$

$$q_{mp} = \alpha(x_m - x_p) + \beta(y_m - y_p);$$
$$r_{mp} = \sqrt{(x_m - x_p)^2 + (y_m - y_p)^2}$$

$\mu(p)$ - unknown density of the potential (13).
If $r \to 0$, McDonald function has log feature

$$K_0(r) = \ln\left(\frac{1}{r}\right) + \varepsilon(r); (14)$$

but if $r \to \infty$,

$$K_0(r) = \sqrt{\frac{\pi}{2r}} e^{-r} + \varepsilon\left(\frac{1}{r}\right). \ (15)$$

Therefore, expression (13) is generalized potential of the type of the potential simple layer. [10]
Now write formal solution of the Poisson equation (9)

$$\psi(m) = \frac{1}{\pi} \iint_{D*} \Omega(p) \cdot \ln(1/rmq) \cdot dS_p + \psi_0; \quad (16)$$

$$\Delta\psi_0 = 0. \quad (17)$$

By applying (15), (13) obtain $\Omega \to 0, r \to \infty$
Boundary conditions (5) for velocity will be rest satisfied if

$$\psi_0 = \alpha y - \beta x. \quad (18)$$

$$\frac{1}{\pi} \iint_{D*} \Omega(p) \cdot \ln(1/rmq) \cdot dS_p \Big|_l = (\beta x - \alpha y)|_l. \quad (19)$$

Now assume that $\mu(m)$ (a solution of the linear integral equation (19), (13) relatively density) can be expanded in Fourier polynomial

$$\mu(\theta) = \sum_{k=0}^{n} (a_k cosk\theta + b_k sink\theta). \quad (20)$$

θ – polar angle,

x=x(θ), y=y(θ) – equations for contour l, $\theta\epsilon[0, 2\pi]$.

Substituting (20) into (13), (16), (19), obtain

$$\mu(x) = \sum_{k=0}^{n} (a_k A_k(m) + b_k B_k(m)), \quad (21)$$

Where

$$A_k(m) = \frac{1}{\pi^2} \iint_{D^*} \ln\left(\frac{1}{r_{mp}}\right) dS_p \int_0^{2\pi} e^{\gamma q_{pQ}} K_0(\gamma r_{PQ}) \times$$
$$\times \sqrt{\dot{x}^2(\theta_Q) + \dot{y}^2(\theta_Q)} \cos(\theta_Q) d\theta_Q;$$

$$B_k(m) = \frac{1}{\pi^2} \iint_{D^*} \ln\left(\frac{1}{r_{mp}}\right) dS_p \int_0^{2\pi} e^{\gamma q_{pQ}} K_0(\gamma r_{PQ}) \times$$
$$\times \sqrt{\dot{x}^2(\theta_Q) + \dot{y}^2(\theta_Q)} \sin(\theta_Q) d\theta_Q. \quad (22)$$

In order to find $A_k(m)$ and $B_k(m)$ break $[0, 2\pi]$ on (2n+1) parts (on number of required coefficients) arbitrarily, for example, points

$$\theta_i = \frac{2\pi}{2n+1} i, i = 0,1, \dots, 2n. \quad (23)$$

Calculating $A_k(\theta_i)$ and $B_k(\theta_i)$, then setting up them into (21) and (19), get system of the linear algebraic equations relative $A_k(\theta_i)$ and $B_k(\theta_i)$ solving which find stream function (21), density of the potential (20), a rotor of velocity (13) and a field of velocities (7).
Calculating div from both parts (6) of that α = const, β = const, and div$\vec{v}$=0, obtain

$$\Delta p = 0 \quad (24)$$

Multiplying (6) scalar by a vector $\vec{n}(n_x; n_y,)$ which is normal to *l*, write down

$$\frac{\partial p}{\partial n}\Big|_l = \Phi(x, y), \quad (25)$$

$$\Phi(x, y)|_l = [\Delta\vec{v} - Re(\vec{u}\vec{\nabla})\vec{v}]\big|_l \vec{u}, \quad (26)$$

The solution of an external regional task of Neumann (24), (26) looks

$$P(m) = P_0 + \frac{1}{\pi} \oint_l \ln\left(\frac{1}{r_{mp}}\right) \chi(p) dl_p, \quad (27)$$

Where density of potential (27) satisfies to Fredholm`s linear integrated equation of the second kind.

$$\pi\chi(m) + \int_0^m \frac{cos\varphi_{mp}}{r_{mp}} \chi(p) dl_p = \Phi(m). \quad (28)$$

φ_{mp}- angle between the normal to l at P and r_{mp}.
Solving (28), (29), find the pressure distribution (27).

II

The resulting solution gives zero approximation of the considered problem in the Oseen approximation.

For his "improvement" write the resulting field of velocities in the form [5]

$$v_x = \alpha + \sum_{k=1}^{m} \frac{\varphi_k(x,y)}{r^k},\ v_y = \beta + \sum_{k=1}^{m} \frac{\delta_k(x,y)}{r^k},$$

$$r = \sqrt{x^2 + y^2} \qquad (1)$$

$\varphi_k, \delta_k -$ known functions.

Linearize the equation (1.4) by (1) and then look for its solution in the form of

$$\Omega = \Omega_0 + \sum_{k=1}^{m} \frac{\Omega_k}{r^k} \quad (2)$$

$$Re\left(\alpha + \sum_{k=1}^{m} \frac{\varphi_k}{r^k}\right)\frac{\partial}{\partial x}\left(\Omega_0 + \sum_{k=1}^{m} \frac{\Omega_k}{r^k}\right) + \quad \left(\beta +\right.$$
k=1mδkrk∂∂yΩ0+k=1mΩkrk=ΔΩ0+k=1mΩkrk(3)

Collecting in (3) the terms of the same powers r, obtain the equation for the decomposition (2). It should be noted that Ω_0 already known (1.13). The equation for Ω_1 is

$$\Delta\Omega_1 - Re\left(\alpha \frac{\partial}{\partial x}\Omega_1 + \beta \frac{\partial}{\partial y}\Omega_1\right) = f_1(x,y); \qquad (4)$$

$$f_1(x,y) = Re\left(\varphi_1 \frac{\partial}{\partial x}\Omega_0 + \delta_1 \frac{\partial}{\partial y}\Omega_0\right). \qquad (5)$$

Consequently,

$$\Omega_1 = \frac{1}{\pi}\oint_l \ e^{\gamma q mp} K_0(\gamma rmp)\mu_1(p)\, dl_p - \frac{1}{\pi}\iint_{D^*} ln\left(\frac{1}{rmp}\right) f_1(p) dS_p. \qquad (6)$$

Solving the Poisson

$$\Delta\psi_1 = -\Omega_1, \qquad (7)$$

finding

$$\psi_1 = \frac{1}{\pi}\iint_{D^*} ln\left(\frac{1}{rmp}\right)\Omega_1(p) dS_p, \qquad (8)$$

and boundary condition

$$\psi_1|_l = 0. \ (9)$$

Since D^* is arbitrary domain (l – anyone sufficiently smooth contour), then from (8) and (9) that $\Omega_1|_l = \theta_{\tau l}$

$$\oint_l \ e^{\gamma q_{mp}} K_0(\gamma r_{mp})\mu_1(p) dl_p\Big|_{m\in l} = \iint_{D^*} ln\left(\frac{1}{rmp}\right) f_1(p) dS_p\Big|_{m\in l} \qquad (10)$$

Thus, obtain a linear integral equation of the first kind of Fredholm type

Solving equation (10), obtain (6), (8) and

$$v_x^{(1)} = \frac{\partial\psi_1}{\partial y}, v_y^{(1)} = -\frac{\partial\psi_1}{\partial x}. \qquad (11)$$

Furthermore, assuming

$$P = P^0 + \sum_{k=1}^{m} \frac{P_k(x,y)}{r^k}, \qquad (12)$$

where $P^0(\mathrm{x,y})$ is known from (1.27), finding $P_1(\mathrm{x,y})$ in the form of

$$P_1 = \frac{1}{\pi}\oint_l \ e^{\gamma q_{mp}}\chi_1(p) dl_p;$$

$$\pi\chi_1(m) + \int_0^m \frac{cos\varphi_{mp}}{r_{mp}}\chi_1(p) dl_p = \Phi_1(m). \qquad (13)$$

Expression $\Phi_1(m)$ contains already known functions Ω_0 , φ_1, ψ_1 and their multiplies.

III

In a case when the field of velocities and pressures (IVF) is considered in bounded domain (for example, in a problem of movement (IVF) in the area due to the motion of part of the boundary of this domain (cavity)) solution procedure considered in sections (1, 2) can be inapplicable.

For example, consider solution of the well-known problem of theory of hydrodynamic greasing [4], when the motion (IVF) is investigated in the area between the two circles of radius R_1 and R_2 off-center due to the rotation of the smaller circle with a given angular velocity ω. (Fig.2.)

Introduce polar coordinates (r, θ) with the center O_1. The equation of the rotating circle is $r=R_1$, the equation of the fixed circle $r=R(\theta)$ is defined by Theorem cosines.

$$R(\theta) = \varepsilon\cos(\theta - \varphi) + \sqrt{R_2^2 - \varepsilon^2 sin^2(\theta - \varphi)} \qquad (1)$$

$$\varepsilon = O_1O_2;$$

φ – angle between the O_1O_2 and the axis Ox (if $\theta = \varphi$, then $r = R_2 + \varepsilon$, if $\theta = \pi + \varphi$ then $r = R_2 - \varepsilon$). Thus $M(r,\theta) \in D$, $r \in [R_2 - \varepsilon, R_2 + \varepsilon]$, $\theta \epsilon [0, 2\pi]$

(2)

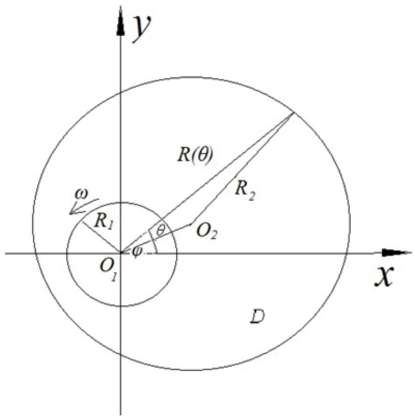

Fig. 2.

In cylindrical coordinates generalized Helmholtz equation is

$$v_r \frac{\partial \Omega}{\partial r} + \frac{v_\theta}{r}\frac{\partial \Omega}{\partial \theta} = \upsilon \Delta \Omega; \qquad (3)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\theta_r) + \frac{\partial v_\theta}{\partial \theta} = 0. \qquad (4)$$

$$\Delta\psi = -\Omega. \qquad (5)$$

Now introduce the dimensionless variables: $r^* = \frac{r}{R_2}$, $\Omega^* = \frac{\Omega}{\omega {R_2}^2}$, $\mathrm{v}^* = \frac{\mathrm{v}}{\omega R_2}$, $p = \rho\omega^2 R_2^2 p^*$ and write

$$Re\left[v_2 \frac{\partial \Omega}{\partial r} + \frac{v_Q}{r}\frac{\partial \Omega}{\partial \theta}\right] = \Delta\Omega. \qquad (6)$$

$$Re = \omega \frac{R_2^2}{\upsilon}, \delta = \frac{R_1}{R_2}.$$

Substituting $v_\theta = \frac{1}{r}, v_r = 0$ (7), linearize (6), that satisfy the continuity equation, and obtain

$$\frac{Re}{r^2}\frac{\partial \Omega}{\partial \theta} = \Delta\Omega \qquad (8)$$

With the substitution

$$\Omega = e^{\gamma\theta}\lambda(r,\theta) \qquad (9)$$

reduce (3.8) to the canonical form

$$\frac{\partial^2 \lambda}{\partial r^2} + \frac{1}{r}\frac{\partial \lambda}{\partial r} + \frac{1}{r^2}\frac{\partial \lambda^2}{\partial \theta^2} - \frac{\gamma^2}{r^2}\lambda = 0, \gamma = \frac{Re}{2} \qquad (10)$$

Solution of equation (10), depending only on r is

$$\lambda = C_1 r^{\gamma} + C_2 r^{-\gamma} \qquad (11)$$

By (7) choose

$$\lambda = C r^{-\gamma} \qquad (12)$$

that allows us to write the integral representation of the solution (8):

$$\Omega(m) = \frac{1}{\pi}\oint_{l_1} e^{\gamma(\theta_m-\theta_p)} r_{mp}^{-\gamma}\,\mu_1(p)dl_p - $$

$$-\frac{1}{\pi}\oint_{l_2} e^{\gamma(\theta_m-\theta_p)} r_{mp}^{-\gamma}\,\mu_2(p)dl_p. \qquad (13)$$

$$r_{mp} = \sqrt{{r_m}^2 + {r_p}^2 - 2r_m r_p \cos(\theta_m - \theta_p)}$$

l_1 – circle radius δ, l_2 – circle radius R_2.

Usually, the section connecting contours of l_1 and l_2 when area D round along its border is made becomes. If the contour of l_1 manages counterclockwise, l_2 – on hour. For this reason there is a minus sign in (13) before integral along l_2.

Let's write down the solution Poisson's equation (5) in a form

$$\psi(m) = \frac{1}{\pi^2}\left[\iint_D ln\left(\frac{1}{r_{mp}}\right)\left[\oint_{l_1} e^{\gamma(\theta_m-\theta_p)} r_{mp}^{-\gamma}\,\mu_1(p)dl_p - \frac{1}{\pi}\oint_{l_2} e^{\gamma(\theta_m-\theta_p)} r_{mp}^{-\gamma}\,\mu_2(p)dl_p\right]\chi_1(p)dS_p\right] + + \frac{1}{\pi}\left[\oint_{l_1} \ln\left(\frac{1}{r_{mp}}\right)\chi_1(p)dl_p - \oint_{l_2} ln\left(\frac{1}{r_{mp}}\right)\chi_2(p)dl_p\right]$$

(14)

Boundary conditions

$$v_r|_{m\in l_1} = 0;\ v_r|_{m\in l_2} = 0;$$

$$v_\theta|_{m\in l_1} = \frac{1}{\delta};\ v_\theta|_{m\in l_2} = 0. \qquad (15)$$

$$v_r = -\frac{1}{r}\frac{\partial\psi}{\partial\theta},\ v_\theta = -\frac{\partial\psi}{\partial r}. \qquad (16)$$

give system of four linear integrated equations concerning density of potentials $\mu_1(m)$, $\mu_2(m)$, $\chi_1(m)$, $\chi_2(m)$.

Rewrite conditions (15) and (16) in a standard form

$$\left.\frac{\partial\Psi}{\partial n}\right|_{r=\delta} = \frac{1}{\delta};\ \left.\frac{\partial\Psi}{\partial n}\right|_{r=R(\theta)} = 0; \qquad (17)$$

$$\left.\frac{\partial\Psi}{\partial\tau}\right|_{r=\delta} = 0;\ \left.\frac{\partial\Psi}{\partial\tau}\right|_{r=R(\theta)} = 0. \qquad (18)$$

Where

$$\left.\frac{\partial\Psi}{\partial n}\right|_{r=\delta} = \left.\frac{\partial\Psi}{\partial r}\right|_{r=\delta};\qquad \left.\frac{\partial\Psi}{\partial n}\right|_{r=R(\theta)} = \left[\frac{\partial\Psi}{\partial r}cos\zeta + \right.$$

1rΨθsinζr=R(θ) (19)

$\frac{\partial \Psi}{\partial \tau}\Big|_{r=\delta} = \frac{1}{r}\frac{\partial \Psi}{\partial \theta}\Big|_{r=\delta}; \frac{\partial \Psi}{\partial \tau}\Big|_{r=R(\theta)} = \Big[\frac{1}{r}\frac{\partial \Psi}{\partial \theta} cos\zeta -$
∂Ψ∂rsinζr=R(θ)

According to the theorem of sine, find

$sin\zeta = \varepsilon \sin(\theta - \varphi)$,

$$cos\zeta = \pm\sqrt{1 - \varepsilon^2 sin^2(\theta - \varphi)} \qquad (20)$$

From (18) follows that stream function accepts constant values on circles $r = \delta$ и r=R(θ). As Ψ is defined to within constant composed, can put

$$\psi|_{r=R(\theta)} = 0 \qquad (21)$$

From (17) and (19) find

$$\psi|_{r=\delta} = -ln\delta. \qquad (22)$$

In other words, stream function is the solution of the equation of Poisson (5) with boundary conditions

$$\frac{\partial \Psi}{\partial n}\Big|_{r=\delta} = \frac{1}{\delta}, \text{ (Neumann's external task)} \qquad (23)$$

$$\frac{\partial \Psi}{\partial n}\Big|_{r=R(\theta)} = 0, \text{ (Neumann's internal task)} \qquad (24)$$

and conditions (21), (22).

Solving these problems, get system of four linear integrated equations

$\iint_D \frac{\partial}{\partial n}\left(ln\frac{1}{r_{mp}}\right)\Omega(p)dS_p + \oint_{l_1} \frac{\partial}{\partial r}\left(\ln\frac{1}{r_{mp}}\right)\chi_1(p)dl_p -$
*l2∂∂r*ln1*rmpχ2pdlp+πχ1m=πδ;m∈l*1; (25)

$\iint_D \frac{\partial}{\partial n}\left(ln\frac{1}{r_{mp}}\right)\Omega(p)dS_p + \oint_{l_1} \frac{\partial}{\partial r}\left(\ln\frac{1}{r_{mp}}\right)\chi_1(p)dl_p -$
*l2∂∂r*ln1*rmpχ2pdlp+πχ2m=*0;*m∈l2*; (26)

$$\psi|_{r=\delta} = -ln\delta; \qquad (27)$$

$$\psi|_{r=R(\theta)} = 0. \qquad (28)$$

In (25) and (26) ψ is defined according to (14).

The solution of this system can be transformed to solution of the system *8n+4* the linear algebraic equations if to look for density of potentials in the form of Fourier's polynomials as it was made in section 1.

Rewrite the linearized Navier – Stokes equations (NS)

$\frac{\partial p}{\partial r} = -Re\left(\frac{1}{r^2}\frac{\partial v_2}{\partial \theta} - \frac{1}{r^2}\right) + \frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} +$

$+\frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta} - -\frac{v_r}{r^2};$

$$\frac{1}{r}\frac{\partial p}{\partial \theta} = -\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{1}{r}\frac{\partial v_\theta}{\partial r} - \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta} - \frac{v_\theta}{r^2}. \qquad (29)$$

Calculating div grad(p), obtain the equation of Poisson in which the right part depends on the solution of the task gotten above

$$\Delta P = f; f = \vec{v}\overrightarrow{\Delta v} - Re\left(\frac{\Delta v^2}{2} - \vec{\Omega}\,\overrightarrow{rot}\,\overrightarrow{v_0}\right) \qquad (30)$$

$\vec{V}_0 = \left(0, \frac{1}{r}\right), \vec{v}(v_r, v_\theta), \vec{\Omega} = (0,0,\Omega)$

$\frac{\partial p}{\partial n}\Big|_{l_1} = \frac{\partial p}{\partial r}\Big|_{l_1},$

$\frac{\partial p}{\partial n}\Big|_{l_2} = \Phi,$

$$\Phi = \frac{\partial p}{\partial r}\Big|_{l_2} cos\zeta + \frac{1}{r}\frac{\partial p}{\partial \theta}\Big|_{l_2} sin\zeta\Big|_{l} \qquad (31)$$

For (27), write down

$\iint_D \left(ln\frac{1}{r_{mp}}\right) f(p)dS_p + \oint_{l_1}\left(\ln\frac{1}{r_{mp}}\right)\varphi_1(p)dl_p -$
*l2*ln*1rmpφ2pdlp*; (32)

Functions $\varphi_1(m)$, $\varphi_2(m)$ are defined from the solution of the linear integrated equations similar (23), (24).

The subsequent approximations can be found similar to the procedure described in section 2 if to put

$v_r = \sum_{k=1}^m w_k\varepsilon^k ; v_\theta = \sum_{k=0}^m z_k\varepsilon^k ;\ \Omega = \sum_{k=0}^m \Omega_k\varepsilon^k;$

$\psi = \sum_{k=0}^m \psi_k\varepsilon^k;\ p = \sum_{k=0}^m p_k\varepsilon^k;$ as ε<<1, it is possible to assume that these decomposition have to tend quickly to achievement given to accuracy.

IV References

[1] C.W.Oseen. Über die Stockssehe Formelund eine verwandze Aufgabe in der hydrodynamik. Arkiv f math., Astr, och Fysik, 6(1910), №29.

[2] C.W.Oseen. Über die Stockssehe Formelund eine verwandze Aufgabe in der hydrodynamik. Arkiv f math., ast, och fysik, G.B. , №29, 1910, 7b, №1, 1933.

[3] C.W.Oseen. Neuere Methoden und Ergebuisser Hydrodynamik. Leipzig, 1927.

[4] Кочин Н.Е., Кибель И.А., Розе Н.В. Теоретическая гидромеханика. М., 1963.

[5] Л.Д. Ландау, В.М. Лифшиц. Гидродинамика. Москва «Наука» 1988г.

[6] F.K.G. Odquist. Über die Randwertaufgaben der Hydrodynamik zäher Flüssigkeiteu. Math.z., 32 b., s. 329, 1910.

[7] Lamb H., On the Uniform Motion of a Sphere through a Viscous Fluid, Phil. Mag., 21 (1911), стр. 120.

[8] Hilding Faxen. Exacte Lösung der Oseen cheu Differential – qleichungen einer zähen Flussigkeit für der Fall Traus latiousbewegung eines zylinders. Nova acta Regial Soc. scient. Upsaliensis, vol. Extra Ordinareu Editum, 1927.

[9] S. Tomotica, T. Aoi. The steady flow of visous fluid a sphere and circular cylinder at small Reynolds number. Quart. Y. Mech. and Appl. Math., v. III, p. 140, 1950, там же V.IV, p. 401, 1951.

[10] О.И. Панич. Решение системы уравнений Озейна для установившегося обтекания плоского контура потоком несжимаемой вязкой жидкости методом потенциалов. Известия высших учебных заведений. Математика. 1962 №3(28), там же, №41962, №6(31), 1962.

[11] О.А. Ладыженская. Математические вопросы динамики несжимаемой вязкой жидкости. «Наука» Москва 1970.

[12] Строчков И.А., Хватцев А.А. Решение задачи поперечного обтекания цилиндра однородным потоком несжимаемой вязкой жидкости (НВЖ) в приближении Озеена. Математические методы в технике и технологиях ММТТ-25: Сборник трудов XXV международной научной конференции. Саратов: Изд-во СГТУ, 2012. – том 1–С. 33 -38.