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OPTIMIZATION OF PARAMETERS OF HEATING ELEMENTS FOR FLOOR PANEL OF PIGLETS RESTING PLACES

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Abstract. *The article deals with problem solving of mathematical physics using the method of separation of variables optimizing heating element – optimizing water tube parameters (tube material, radius, insulation thickness, choosing appropriate surrounding environment). For ensuring piglets comfort, concrete floor panels heated by electric current or hot water are used. If an electro-heated cable in the panels body is placed, than amount of heat conducted from the cable is the same along all the length of the cable. If hot water circulating through tube is used, than amount of heat energy taken off the heater decreases along its length. The aim of the research is to create the mathematical model of a water tube, were water temperature is gradually decreasing. This model is needed to make precise calculations of the heating panel for piglets, to ensure equal temperature distribution over the upper surface of the panel, taking into account mathematically calculated temperature decrease of the heat source.*

Keywords: *mathematical modeling, heated floors, water tube.*

Introduction

New born piglets together with the sow are kept. The optimal surrounding air temperature for a sow is about 16…20°C, but for new born piglet during the first days of their life the temperature in its lairs has to be within the limits of 32…36°C. Gradually the lair's temperature must be decreased until 22…24°C when piglets are two months of age and weaned [Priekulis et al., 1992]. That means that comfortable surrounding temperature for sows and piglets is different despite the fact that during first days they are kept together. Therefore in cold winter countries like Latvia piglets resting place local warming ought to be used.

For local warming heating panels are usually used. They can be made from different materials and using different production technologies. The energetic efficiency of the panels will result from the integrated insulation which minimizes the downward heat emission and from the even distribution over the panel. To improve evenness of the temperature distribution it is necessary to make precise mathematical model of the heating panel and then to check it experimentally. Nowadays very popular are plastic pads which are filled in with water, but the evenness of the temperature is not so good [MIK, 2009], as it can be achieved using tubes with hot water placed into some kind of solid material, in our examples we used data for concrete, but there is a possibility to use another materials (with lesser abrasive properties) [Zoric. M. et al., 2009]. The energy consumption of the calculated concrete panel is lower than the most popular panels in the market is as well, comparing 320 W m^{-2} [MIK, 2009], to 290W m^{-2} at the same working temperatures. In terms of energy cost this leads to the enormous energy economy and decrease of CO_2 pollutions as well.

Materials and methods

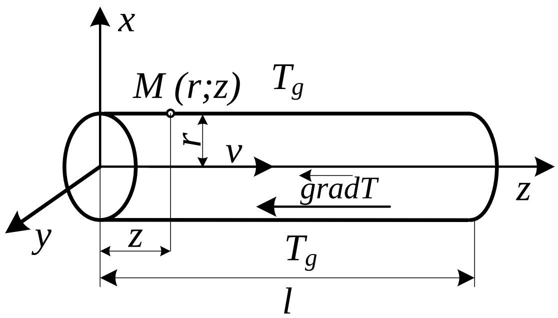

Fig.1. **Pipe element for calculation of temperature drop over the fluid flow in the tube**

Assuming, that tube is being bathed by environment, which temperature T_0 and flowing into the tube fluid temperature $T_1>T_0$ (the fluid gives the heat energy to he surrpunding environment). These processes can be described with equation (1) [Riekstins, 1969]:

$$a\Delta T - \vec{v} gradT = 0. \quad (1)$$

For concrete problem formulation it is necessary to give boundary conditions. According to our task, the fluid flows into the tube with temperature T_1 (2):

$$T|_{z=0} = T_1. \quad (2)$$

Conditionally we can accept that outflow fluid temperature is equal to surrounding air temperature (3):

$$T|_{z\to\infty} = T_0. \quad (3)$$

It is necessary to formulate the third type boundary conditions as well for the tube surface (4).

$$-\lambda \frac{\partial T}{\partial r}\Big|_{r=R} = \alpha(T|_{r=R} - T_0), \quad (4)$$

where

α- heat transfer coefficient, W m^{-2} K^{-1};

λ_2 - thermal conductivity, W (m K)$^{-1}$.

Now we have defined the problem of mathematical physics, which contains equation (1) and border conditions (2-4). For solving such a problem, we will use the method of separation of variables. In cylindrical coordinate system the Laplace operator is written in a form:

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2} = 0. \quad (5)$$

So, equation (6) is a stationary equation, which contains conditions of fluid flow:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) - \frac{\vec{v}}{a}\frac{\partial T}{\partial z} + \frac{\partial^2 T}{\partial z^2} = 0. \quad (6)$$

The solution of the problem we will look for as sum of endless row (7):

$$T(r,z) = T_0 + \sum_k R_k(r) \cdot Z_k(r). \quad (7)$$

Inserting representation (7) into the basic equation (6), accordingly substituting $\frac{\partial R_k}{\partial r} = R'_k$ and $\frac{\partial Z_k}{\partial z} = Z'_k$.we will get the following:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial(T_0+\sum_k R_k(r)Z_k(z))}{\partial r}\right) - \frac{\vec{v}}{a}\frac{\partial(T_0+\sum_k R_k(r)Z_k(z))}{\partial z} + \frac{\partial^2(T_0+\sum_k R_k(r)Z_k(z))}{\partial z^2} = \\ = \frac{1}{r}\frac{\partial}{\partial r}\sum_k \left(r\cdot(R_k{}'\cdot Z_k)\right)' - \sum_k \frac{v}{a}R_k Z_k{}' + \sum_k R_k Z_k{}'' = \\ \sum_k \frac{1}{r}Z_k(R_k{}' + rR_k{}'') - \frac{v}{a}R_k Z_k{}' + R_k Z_k{}'' = 0 \quad (8)$$

The solution of the equation (8) is found only in case, when each member of the sum row is equal with zero:

$$\frac{1}{r}Z_k\left(R_k{}' + rR_k{}''\right) - \frac{v}{a}R_k Z_k{}' + R_k Z_k{}'' = 0 \quad (9)$$

Now, dividing equation (9) with$R_k Z_k$, we can get equation were the right side is dependant from one argument, and the left side from another argument (10):

$$\frac{1}{r}\frac{R_k{}'}{R_k} + \frac{R_k{}''}{R_k} = \frac{v}{a}\cdot\frac{Z_k{}'}{Z_k} + \frac{Z_k{}''}{Z_k}. \quad (10)$$

To get 2 equations with one unknown, we need to compare it with free chosen constant μ_k^2 (11):

$$\begin{cases} \frac{1}{r}\frac{R_k{}'}{R_k} + \frac{R_k{}''}{R_k} = \mu_k^2 \\ \frac{v}{a}\cdot\frac{Z_k{}'}{Z_k} + \frac{Z_k{}''}{Z_k} = \mu_k^2 \end{cases} \quad (11)$$

After algebraic changes we obtain two second order differential equations (12):

$$\begin{cases} rR_k{}'' + R_k{}' + \mu_k^2 rR_k = 0 \\ Z_k{}'' + \frac{v}{a}\cdot Z_k{}' + \mu_k^2 Z_k = 0 \end{cases} \quad (12)$$

The first system's (12a) equation is special case of the general equation (13) which is called Bessel equation:

$$r^2 R''_k + r\,R'_k \pm \left(\mu_k^2 r^2 - \nu^2\right)R_k = 0, \quad (13)$$

where ν is numeral parmeter, in our case $\nu = 0$.

The solution of the Bessel equation is expressed with Bessel functions $J_\nu\,(\mu_k r)$. According to it the solution of the equation (12a) is:

$$R_k(r) = A_k\cdot J_0\left(\mu_k r\right) + B_k\cdot Y_0\left(\mu_k r\right), \quad (14)$$

$$Y_0 = 0 \implies B_k\cdot\left(\mu_k r\right) = 0, \quad (15)$$

$$R_k(r) = A_k\cdot J_0\left(\mu_k r\right), \quad (16)$$

where A_k is free chosen integration constant;

$J_0(\mu_k r)$ - Bessel function, when parameter $\nu = 0$.

The equation (12b) is the 2nd order differential equation with constant coefficients, dividing it with Z_k, and making substitution $\frac{vc\rho}{\lambda} = 2\beta$ it is written in the following form (17):

$$Z_k''(z) - 2\beta Z_k'(z) - \mu_k^2 Z_k(z) = 0. \tag{17}$$

The general solution of the equation (17) is:

$$Z_k(z) = C_k e^{\xi_1 z} + D_k e^{\xi_2 z}, \tag{18}$$

where C_k and D_k are free chosen integration constants,
ξ- particular value.

$$\xi_1 = \beta + \sqrt{\beta^2 + \mu_k^2} \tag{19}$$

$$\xi_2 = \beta - \sqrt{\beta^2 + \mu_k^2} \tag{20}$$

Further process for solution finding is being made: the expressions (18) and (16) are inserted into the equation (7), the equation (21) is obtained. The equation (21) is being inserted into the boundary conditions (2, 3) in consecutive order. To satisfy boundary conditions (2), no restrictions are introduced. The boundary conditions (3) can be satisfied, only when $z \to \infty$, then $e^{(\beta+\sqrt{\beta^2+\mu_k^2})z} \to \infty$, it means, that we have to assume $C_k = 0$, to satisfy boundary conditions (3). The following is obtained:

$$T(r;z) = T_0 + \sum_k J_0(\mu_k r)\left(D_k e^{(\beta-\sqrt{\beta^2+\mu_k^2})z}\right), \tag{21}$$

where -D_k is free chosen constant $A_k \cdot D_k$
As we see from equation (21) we do not know free chosen constants μ_k and D_k.
To determine free chosen constants μ_k we will use side surface boundary conditions (4), the equation (21) is inserted into the boundary conditions (4):

$$-\lambda \sum_k D_k\left[-J_1(\mu_k R)\right] \cdot \mu_k \cdot e^{\left(\beta-\sqrt{\beta^2+\mu_k^2}\right)z} = \alpha\left(T_0 + \sum_k D_k \cdot [J_0(\mu_k R)] \cdot e^{(\beta-\sqrt{\beta^2+\mu_k^2})z} - T_0\right). \tag{22}$$

After row of algebraically changes we obtain:

$$\mu_k = \frac{\alpha}{\lambda} \cdot \frac{J_0(\mu_k R)}{J_1(\mu_k R)}. \tag{23}$$

As Bessel function argument is dimensionless, than $[\mu_k] = [m^{-1}]$; to introduce into the solution Biot number $\frac{\alpha}{\lambda} \cdot R = b$, we need to multiply equation (23) with R:

$$\mu_k R = \frac{\alpha}{\lambda} \cdot R \cdot \frac{J_0(\mu_k R)}{J_1(\mu_k R)}. \tag{24}$$

We will mark $\mu_k R$ as X_k:

$$X_k = b \cdot \frac{J_0(\mu_k R)}{J_1(\mu_k R)}. \tag{25}$$

So we have got the transcendental equation for particular value determination.

Now it is needed to determineD_k, to obtain it we will put general equation (21) into the boundary conditions (2), when $z \to 0$:

$$T(r;0) = T_1 = T_0 + \sum_k J_0(\mu_k r) \cdot D_k e^{\left(\beta - \sqrt{\beta^2 + \mu_k^2}\right)z} = \\ = T_0 + D_1 J_0(\mu_1 r) + D_2 J_0(\mu_2 r) + D_3 J_0(\mu_3 r) + \cdots + D_k J_0(\mu_k r). \quad (26)$$

Using properties of the particular functions' scalar multiplication multiplying with $rJ_0(\mu_k r)$ and integrating, we obtain (27):

$$\int_0^R rJ_0(\mu_k R)(T_1 - T_0)dr = D_k \int_0^R rJ_0{}^2(\mu_k r)^2 dr. \quad (27)$$

The equation above enables to get solution for D_k as division of two integrals:

$$(T_1 - T_0)\frac{\int_0^R rJ_0(\mu_k r)dr}{\int_0^R rJ_0{}^2(\mu_k r)^2 dr} = D_k. \quad (28)$$

Now we need to develop function into a row (29), where trigonometrically functions are taken for Furje rows:

$$1 = \sum_k C_k J_0(\mu_k r), \quad (29)$$

where - C_k is Furje coefficient.
From equation (28) it is seen, that:

$$D_k = (T_1 - T_0)C_k; \quad (30)$$

$$f(r) = \sum_k C_k J_0(\mu_k r), \quad (31)$$

where $f(r)$ – whatever function, which can be impulse function as well.
Now we need both equations (29 and 30) to multiply with $rJ_0(\mu_k r)$ and integrate from “0” to “R”.
The particular function I_k is obtained:

$$I_k = \frac{R^2}{2}\left(J_0{}^2(\mu_k R) + J_1{}^2(\mu_k R)\right). \quad (32)$$

and

$$C_k = \frac{2J_1(\mu_k R)}{\mu_k R\left[J_0{}^2(\mu_k R) + J_1{}^2(\mu_k r)\right]}. \quad (33)$$

Results and discussion

The solution for determination the temperature drop during the fluid flow through the tube is obtained:

$$T(r;z) = T_0 + 2(T_1 - T_0)\sum_{k=1}^{\infty} \frac{J_1(\mu_k R) \cdot J_0(\mu_k r) \cdot e^{(\beta - \sqrt{\beta^2 + \mu_k^2})z}}{(\mu_k R)\left[J_0{}^2(\mu_k R) + J_1{}^2(\mu_k R)\right]}. \quad (34)$$

According to this equation, it is possible to calculate temperature drop during fluid flow through the tube at the different conditions. In the figure 1, different cases of the fluid flow are presented, three curves are being compared, when diameter of the tube and heat transfer coefficient is changeable, and other parameters remain constant.
From the picture it is seen, that temperature decrease of the fluid (in our case water) is exponentially dependant from the tube length. Three cases for simulation were chosen (diameter value 1 cm, 1.5 cm, 2 cm). In the first case temperature drop during the 1 m fluid flow through the tube is 1.01%, in the second and third cases 0.67% and 0.50% respectively. The value of the heat transfer coefficient was changed theoretically, but further it is necessary to make train of experiments to determine heat transfer coefficient dependency on tube

diameter and other important parameter as flow speed, fluid type and surrounding environment.

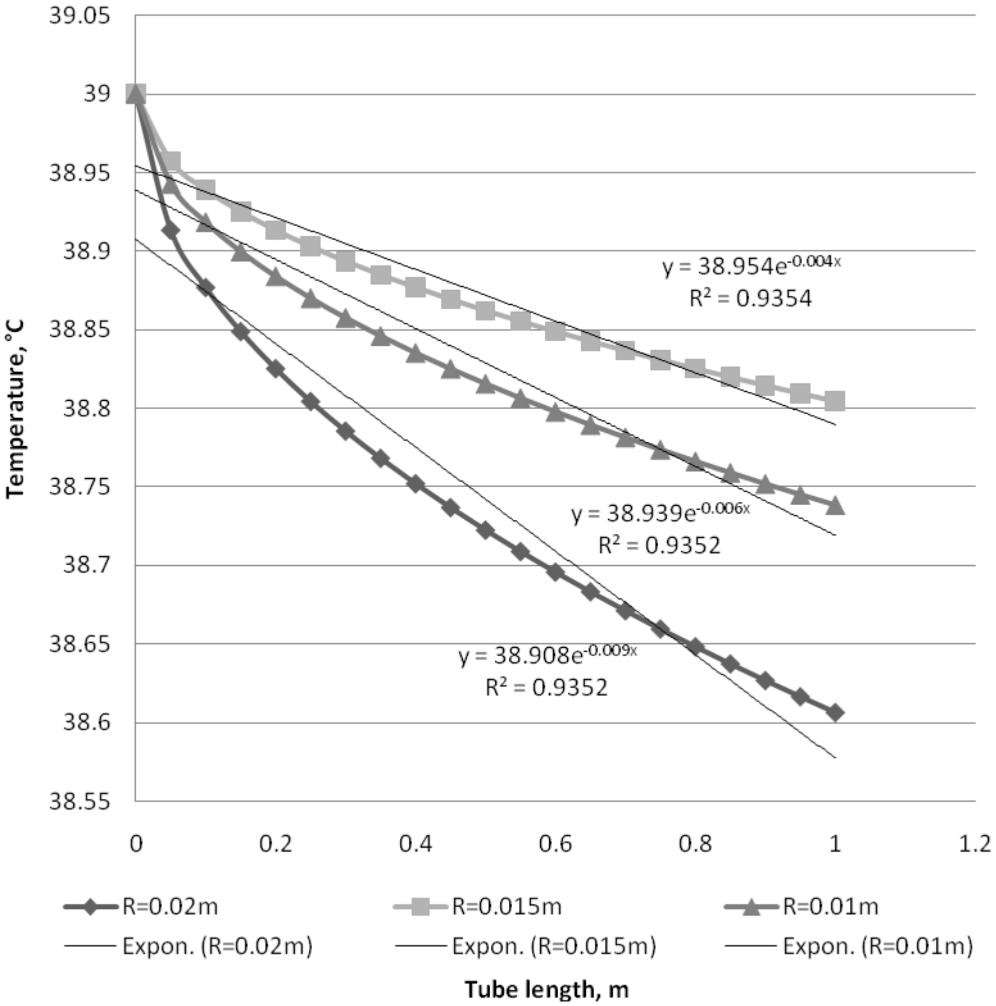

Fig. 1. **Fluid flow through the water tube with different diameters**

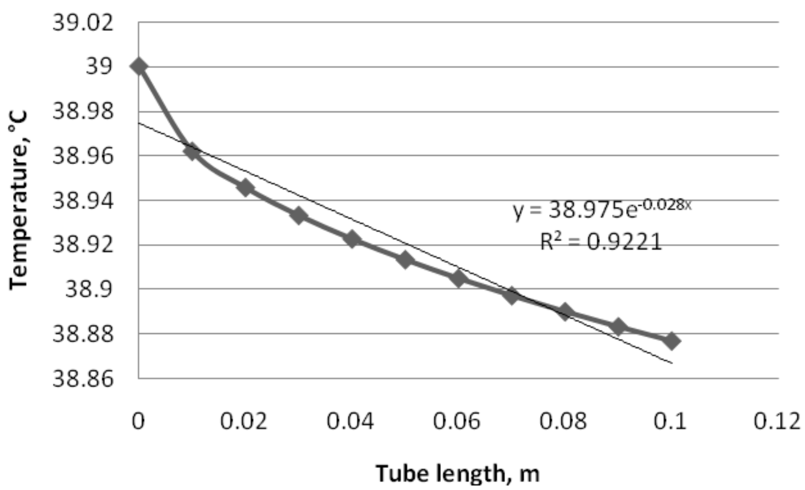

Fig. 2. **Temperature drop of the fluid during the first 10 cm flow**

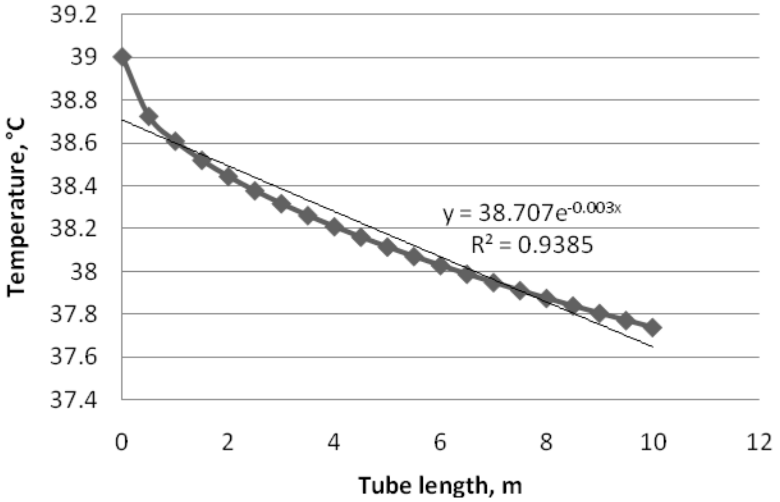

Fig. 3. **Temperature drop of the fluid during the 10 m flow**

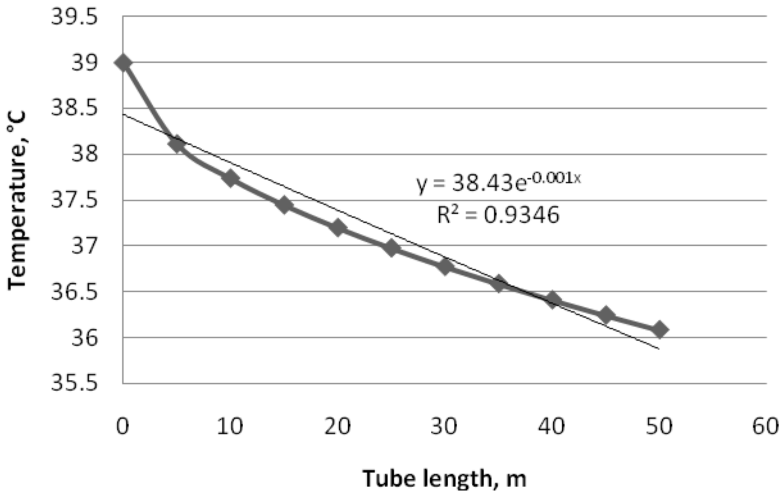

Fig. 4. **Temperature drop of the fluid during the 50 m flow**

Comparing figures two, three and four. We can see the tendency of the temperature drop depending on the fluid flow distance, for example, when fluid flows 10 cm the temperature drop of 0.32% occurs, but when distance is ten times greater, the temperature drop only for 1.01% appears, when we speak about longer distances, then at 50 m distance it will be 3.24%. Using obtained formula it is possible to calculate the temperature drop of fluid at different fluid flow distances. Using previously obtained formulas [Zagorska V., et al. 2010], it is possible to make the model of the heating panel for various materials, flow rates, surrounding environment parameter values.

Conclusions

1. Formulas (33), (34) and (25) make the mathematical model of a tube with fluid flowing inside it at boundary conditions (2) to (4).
2. They enable to calculate the temperature drop, during the fluid flow at certain values of the parameters included into the formulas.
3. These calculations enable to analyze coherence among different parameters of a water tube.

4. It is necessary to determine heat transfer coefficient dependence on the tube diameter, fluid flow speed and surrounding environment experimentally, using well known equations.

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